Math 112 – Review for Test 1

Directions: Show your work for full credit. The way you drive your answer is most important.

1) Let $f(x) = x + \sin(x)$. Estimate the value of $\int_0^3 f(x)dx$ by evaluating the left sum with three equal intervals. Find the actual value of the integral using the Fundamental Theorem of Calculus, compare your approximation and find the error committed by L_3.

2) Find a definite integral that is approximated by the left sum: $\frac{2}{5}\Sigma_{i=0}^{19} e^{\frac{2}{5}i}$.

3) Use sigma notation to express

1. the left sum approximation to $\int_0^{2\pi} sin(x)dx$
   (a) with 8 equal subintervals
   (b) with n equal subintervals
2. the midpoint sum approximation to $\int_1^3 \frac{1}{1+x^2}dx$
   (a) with 8 equal subintervals
   (b) with n equal subintervals

4) Give an example of a function f and an interval $[a, b]$ so that $L_5 < R_5$.

5) Give an example of a function f and an interval $[a, b]$ so that $M_{10} < L_{10}$.

6) Give an example of a function f and an interval $[a, b]$ so that $M_4 < R_4$.

7) Find the exact value of the integral $\int_1^2 (x^2 + 1)dx$ by setting up a right Riemann sum using sigma notation and using the definition of Riemann integral.

8) Do four steps (step size 0.5) in approximating a solution to the initial value problem $y' = y - x$, $y(0) = 2$ using Euler's method.

9 Let $I = \int_0^3 f(x)dx$.

1. Write the left sum which approximates I with 20 subdivision using sigma notation.
2. The approximation using L_{20} is too ....(small or large) because...
3. Calculate an error bound using the formula $|I - Ln| \leq \frac{K_1(b-a)^2}{2n}$.
4. How many subintervals do guarantee an error committed by L_n of less than 0.0001.

10) Let $I = \int_0^{2\pi} \cos(2x)dx$. Find n large enough so that $|I - T_n| \leq 0.1$.

11) Let $I = \int_a^b f(x)dx$, where f is positive and increasing over the interval $[a, b]$. Indicate whether or not the following statements must be true, cannot be true or maybe true, for all $n \geq 1$.

1. $L_n < I$
2. $M_n < I$
3. $L_n < R_n$
4. $L_n < M_n$
5. $L_n < T_n$
6. $T_n < M_n$.

12) Suppose that $f'(x) > 0$ and $f''(x) > 0$ if $a \leq x \leq b$. Is R_n a more accurate estimate of $I = \int_a^b f(x)dx$ than T_n.

13) Evaluate $\int_1^4 xdx$ by computing the limit of midpoint sums M_n.

14) Evaluate $\lim_{n\to\infty} \frac{1}{n}\Sigma_{j=1}^{n} \left(\frac{j}{n}\right)^3$ by expressing it as a definite integral and then evaluating this integral using the fundamental theorem of calculus.

15) Is $f(0)\cdot 1 + f(2)\cdot 2 + f(4)\cdot 3 + f(6)\cdot 4$ a Riemann sum approximation to $\int_0^{10} f(x)dx$?